International Journal of Theoretical Physics, Vol. 40, No. 7, 2001

Asymmetry of Hawking Radiation of Dirac Particles in a Charged Vaidya–de Sitter Black Hole

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Received December 6, 2000

The Hawking radiation of Dirac particles in a charged Vaidya–de Sitter black hole is investigated by using the method of generalized tortoise coordinate transformation. It is shown that the Hawking radiation of Dirac particles does not exist for P_1, Q_2 components, but for P_2, Q_1 components it does. Both the location and the temperature of the event horizon change with time. The thermal radiation spectrum of Dirac particles is the same as that of Klein-Gordon particles.

1. INTRODUCTION

Hawking's investigation of quantum effects (Hawking, 1974, 1975) interpreted as the emission of a thermal spectrum of particles by a black hole event horizon sets a significant landmark in black hole physics. In the last few decades, much work has been done on the Hawking effect of black holes in different types of space–time, such as Vaidya (Kim *et al.*, 1989), Kerr–Newman (Wu and Cai, 2000a,b), and NUT–Kerr–Newman–de Sitter (Ahmed, 1991; Ahmed and Mondal, 1995) space–times. The thermal radiation of Dirac particles, especially with the aid of Newman–Penrose formalism (Newman and Penrose, 1962), has been also investigated in some spherically symmetric and nonstatic black holes (Zhao *et al.*, 1994; Li and Zhao, 1993; Ma and Yang, 1993; Zhu *et al.*, 1994; Zhang *et al.*, 1999). However, most of these studies concentrated on the spin state $p = 1/2$ of the four-component Dirac spinors. Recently, the Hawking radiation of Dirac particles of spin state $p = -1/2$ attracts a little more attention (Li *et al.*, 1999; Li, 1998; Li and Zhao, 1998).

In this paper, we investigate the Hawking effects of Dirac particles in the Vaidya-type black hole by means of the generalized tortoise transformation method. We consider simultaneously the limiting forms of the first order form and the second

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order form of Dirac equation near the event horizon because the Dirac spinors should satisfy both of them. From the former, we can obtain the event horizon equation, while from the latter, we can derive the Hawking temperature and the thermal radiation spectrum of electrons. Our results are in accord with others. With our new method, we can prove rigorously that the Hawking radiation does not exist for P_1, Q_2 components of Dirac spinors. The origin of this asymmetry of the Hawking radiation of different spinorial components probably stem from the asymmetry of space–time in the advanced Eddington–Finkelstein coordinate system. As a byproduct, we point out that there could not have been any new quantum thermal effect (Li *et al.*, 1999; Li, 1998; Li and Zhao, 1998) in the Hawking radiation of Dirac particles in any spherically symmetric black hole whether it is static or nonstatic.

The paper is organized as follows: In section 2, we work out the spinor form of Dirac equation in the Vaidya-type black hole, then, we obtain the event horizon equation in section 3. The Hawking temperature and the thermal radiation spectrum are derived in section 4 and 5, respectively. Section 6 is devoted to some discussions.

2. DIRAC EQUATION

The metric of a charged Vaidya–de Sitter black hole with the cosmological constant Λ is given in the advanced Eddington–Finkelstein coordinate system by

$$ds^2 = 2\,dv(G\,dv - dr) - r^2(d\theta^2 + \sin^2\theta\,d\varphi^2), \tag{1}$$

and the electromagnetic one-form is

$$\boldsymbol{A} = \frac{Q}{r}\,dv \tag{2}$$

where $2G = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^4$, in which both mass $M(v)$ and electric charge $Q(v)$ of the hole are functions of the advanced time v.

We choose such a complex null-tetrad $\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \overline{\boldsymbol{m}}\}$ that satisfies the orthogonal conditions $\boldsymbol{l}\cdot\boldsymbol{n} = -\boldsymbol{m}\cdot\overline{\boldsymbol{m}} = 1$. Thus the covariant one-forms can be written as

$$\boldsymbol{l} = dv, \qquad \boldsymbol{m} = \frac{-r}{\sqrt{2}}(d\theta + i\sin\theta\,d\varphi),$$

$$\boldsymbol{n} = G\,dv - dr, \qquad \overline{\boldsymbol{m}} = \frac{-r}{\sqrt{2}}(d\theta - i\sin\theta\,d\varphi). \tag{3}$$

and their corresponding directional derivatives are

$$D = -\frac{\partial}{\partial r}, \qquad \delta = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\,\frac{\partial}{\partial\varphi}\right),$$

$$\Delta = \frac{\partial}{\partial v} + G\frac{\partial}{\partial r}, \qquad \overline{\delta} = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\,\frac{\partial}{\partial\varphi}\right). \tag{4}$$

It is not difficult to determine the 12 Newman–Penrose complex coefficients (Newman and Penrose, 1962) in the above null-tetrad:

$$\tilde{\kappa} = \tilde{\lambda} = \tilde{\pi} = \sigma = \epsilon = \tau = \tilde{\nu} = 0, \qquad \rho = \frac{1}{r},$$
$$\mu = \frac{G}{r}, \qquad \gamma = -\frac{G_{,r}}{2}, \qquad \beta = -\alpha = \frac{\cot\theta}{2\sqrt{2}r}. \tag{5}$$

Inserting for the following relations among the Newman–Penrose spin-coefficients[3]

$$\epsilon - \rho = -\frac{1}{r}, \qquad \tilde{\pi} - \alpha = \frac{\cot\theta}{2\sqrt{2}r},$$
$$\mu - \gamma = \frac{G}{r} + \frac{G_{,r}}{2}, \qquad \beta - \tau = \frac{\cot\theta}{2\sqrt{2}r}, \tag{6}$$

and the electromagnetic potential

$$\boldsymbol{A}\cdot\boldsymbol{l} = 0, \qquad \boldsymbol{A}\cdot\boldsymbol{n} = Q/r, \qquad \boldsymbol{A}\cdot\boldsymbol{m} = -\boldsymbol{A}\cdot\overline{\boldsymbol{m}} = 0, \tag{7}$$

into the spinor form of the coupled Chandrasekhar–Dirac equation (Chandrasekhar, 1983), which describes the dynamic behavior of spin-1/2 particles, namely

$$(D + \epsilon - \rho + iq\boldsymbol{A}\cdot\boldsymbol{l})\,F_1 + (\overline{\delta} + \tilde{\pi} - \alpha + iq\boldsymbol{A}\cdot\overline{\boldsymbol{m}})F_2 = \frac{i\mu_0}{\sqrt{2}}G_1,$$
$$(\Delta + \mu - \gamma + iq\boldsymbol{A}\cdot\boldsymbol{n})\,F_2 + (\delta + \beta - \tau + iq\boldsymbol{A}\cdot\boldsymbol{m})F_1 = \frac{i\mu_0}{\sqrt{2}}G_2,$$
$$(D + \epsilon^* - \rho^* + iq\boldsymbol{A}\cdot\boldsymbol{l})\,G_2 - (\delta + \tilde{\pi}^* - \alpha^* + iq\boldsymbol{A}\cdot\boldsymbol{m})\,G_1 = \frac{i\mu_0}{\sqrt{2}}F_2,$$
$$(\Delta + \mu^* - \gamma^* + iq\boldsymbol{A}\cdot\boldsymbol{n})\,G_1 - (\overline{\delta} + \beta^* - \tau^* + iq\boldsymbol{A}\cdot\overline{\boldsymbol{m}})\,G_2 = \frac{i\mu_0}{\sqrt{2}}F_1, \tag{8}$$

where μ_0 and q is the mass and charge of Dirac particles, one obtains

$$-\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)F_1 + \frac{1}{\sqrt{2}r}\mathcal{L}F_2 = \frac{i\mu_0}{\sqrt{2}}G_1, \qquad \frac{1}{2r^2}\mathcal{D}F_2 + \frac{1}{\sqrt{2}r}\mathcal{L}^\dagger F_1 = \frac{i\mu_0}{\sqrt{2}}G_2,$$
$$-\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)G_2 - \frac{1}{\sqrt{2}r}\mathcal{L}^\dagger G_1 = \frac{i\mu_0}{\sqrt{2}}F_2, \qquad \frac{1}{2r^2}\mathcal{D}G_1 - \frac{1}{\sqrt{2}r}\mathcal{L}G_2 = \frac{i\mu_0}{\sqrt{2}}F_1, \tag{9}$$

[3] Here and hereafter, we denote $G_{,r} = dG/dr$, etc.

in which we have defined operators

$$\mathcal{D} = 2r^2\left(\frac{\partial}{\partial v} + G\,\frac{\partial}{\partial r}\right) + (r^2G)_{,r} + 2iqQr,$$

$$\mathcal{L} = \frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}, \qquad \mathcal{L}^\dagger = \frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}.$$

One can observe that the Chandrasekhar–Dirac equation (8) could be satisfied by identifying Q_1, Q_2, qQ with P_2^*, $-P_1^*$, $-qQ$, respectively. By substituting

$$F_1 = \frac{1}{\sqrt{2}r}P_1, \qquad F_2 = P_2, \qquad G_1 = Q_1, \qquad G_2 = \frac{1}{\sqrt{2}r}Q_2,$$

into Eq. (9), they have the form

$$-\frac{\partial}{\partial r}P_1 + \mathcal{L}P_2 = i\mu_0 rQ_1, \qquad \mathcal{D}P_2 + \mathcal{L}^\dagger P_1 = i\mu_0 rQ_2,$$

$$-\frac{\partial}{\partial r}Q_2 - \mathcal{L}^\dagger Q_1 = i\mu_0 rP_2, \qquad \mathcal{D}Q_1 - \mathcal{L}Q_2 = i\mu_0 rP_1. \qquad (10)$$

3. EVENT HORIZON

Now separating variables to Eq. (10) as

$$P_1 = R_1(v,r)S_1(\theta,\varphi), \qquad P_2 = R_2(v,r)S_2(\theta,\varphi),$$

$$Q_1 = R_2(v,r)S_1(\theta,\varphi), \qquad Q_2 = R_1(v,r)S_2(\theta,\varphi),$$

then we have the radial part

$$\frac{\partial}{\partial r}R_1 = (\lambda - i\mu_0 r)R_2, \qquad \mathcal{D}R_2 = (\lambda + i\mu_0 r)R_1, \qquad (11)$$

and the angular part

$$\mathcal{L}^\dagger S_1 = -\lambda S_2, \qquad \mathcal{L}S_2 = \lambda S_1, \qquad (12)$$

where $\lambda = \ell + 1/2$ is a separation constant. Both functions $S_1(\theta,\varphi)$ and $S_2(\theta,\varphi)$ are, respectively, spinorial spherical harmonics ${}_sY_{\ell m}(\theta,\varphi)$ with spin-weight $s = \pm 1/2$, satisfying the following equation by Goldberg *et al.* (1968)

$$\left[\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial}{\partial\varphi^2} + \frac{2is\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - s^2\cot^2\theta + s + (\ell - s)(\ell + s + 1)\right]{}_sY_{\ell m}(\theta,\varphi) = 0. \qquad (13)$$

As to the thermal radiation, we should be concerned about the behavior of the radial part of Eq. (11) near the horizon only. Because the Vaidya-type space–times are spherically symmetric, we introduce as a working ansatz the generalized tortoise coordinate transformation (Zhao and Dai, 1991)

$$r_* = r + \frac{1}{2\kappa} \ln[r - r_{\mathrm{H}}(v)], \qquad v_* = v - v_0, \tag{14}$$

where r_{H} is the location of the event horizon, κ is an adjustable parameter and is unchanged under tortoise transformation. The parameter v_0 is an arbitary constant. From (14) we can deduce some useful relations for the derivatives as follows:

$$\frac{\partial}{\partial r} = \left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right] \frac{\partial}{\partial r_*}, \qquad \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{r_{\mathrm{H},v}}{2\kappa(r - r_{\mathrm{H}})} \frac{\partial}{\partial r_*}.$$

Now let us consider the asymptotic behavior of R_1, R_2 near the event horizon. Under the transformation (14), Eq. (11) can be reduced to the following limiting form near the event horizon

$$\frac{\partial}{\partial r_*} R_1 = 0, \qquad 2r_{\mathrm{H}}^2[G(r_{\mathrm{H}}) - r_{\mathrm{H},v}] \frac{\partial}{\partial r_*} R_2 = 0, \tag{15}$$

after being taken limits $r \to r_{\mathrm{H}}(v_0)$ and $v \to v_0$.

From Eq. (15), we know that $R_1(r_*) = \text{const}$ is regular on the event horizon. Thus the existence condition of a nontrial solution we can have for R_2 is (as for $r_{\mathrm{H}} \neq 0$)

$$2G(r_{\mathrm{H}}) - 2r_{\mathrm{H},v} = 0. \tag{16}$$

which determines the location of horizon. The event horizon equation (16) can be inferred from the null hypersurface condition, $g^{ij} \partial_i F \partial_j F = 0$, and $F(v, r) = 0$, namely $r = r(v)$. It follows that r_{H} depends on time v. So the location of the event horizon and the shape of the black hole change with time.

4. HAWKING TEMPERATURE

In the preceding section, we have deduced the event horizon equation from the limiting form of the separated radial part of the first order Dirac equation. Using a similar procedure to its second order equation, we can derive the Hawking temperature and the thermal radiation spectrum. A direct calculation gives the second order radial equation

$$2r^2 \left(G \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial v \partial r} \right) R_1 + \left[(r^2 G)_{,r} + 2iqQr + 2r^2 G \frac{i\mu_0\lambda - \mu_0^2 r}{\lambda^2 + \mu_0^2 r^2} \right] \frac{\partial}{\partial r} R_1 - \left(\lambda^2 + \mu_0^2 r^2\right) R_1 = 0, \tag{17}$$

$$2r^2\left(G\,\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial v\partial r}\right)R_2+[3(r^2G)_{,r}+2iqQr]\,\frac{\partial}{\partial r}R_2+4r\frac{\partial}{\partial v}R_2$$
$$-\frac{i\mu_0\lambda+\mu_0^2r}{\lambda^2+\mu_0^2r^2}\left[2r^2G\,\frac{\partial}{\partial r}+\,2r^2\,\frac{\partial}{\partial v}+(r^2G)_{,r}+2iqQr\right]R_2$$
$$+\left[(r^2G)_{,rr}+2iqQ-\left(\lambda^2+\mu_0^2r^2\right)\right]R_2=0. \tag{18}$$

Given the transformation (14), Eqs. (17) and (18) have the following limiting forms near the event horizon $r=r_{\rm H}$

$$\left[\frac{A}{2\kappa}+4G(r_{\rm H})-2r_{{\rm H},v}\right]\,\frac{\partial^2}{\partial r_*^2}R_1+2\,\frac{\partial^2}{\partial r_*\,\partial v_*}R_1$$
$$+\left[-A+G_{,r}\,(r_{\rm H})+\frac{2iqQ+2G(r_{\rm H})}{r_{\rm H}}+2r_{\rm H}^2G(r_{\rm H})\frac{i\mu_0\lambda-\mu_0^2r_{\rm H}}{r_{\rm H}^2\left(\lambda^2+\mu_0^2r_{\rm H}^2\right)}\right]\,\frac{\partial}{\partial r_*}R_1$$
$$=\left[\frac{A}{2\kappa}+2G(r_{\rm H})\right]\,\frac{\partial^2}{\partial r_*^2}R_1+2\,\frac{\partial^2}{\partial r_*\,\partial v_*}R_1=0, \tag{19}$$

$$\left[\frac{A}{2\kappa}+4G(r_{\rm H})-2r_{{\rm H},v}\right]\,\frac{\partial^2}{\partial r_*^2}R_2+2\,\frac{\partial^2}{\partial r_*\,\partial v_*}R_2$$
$$+\left\{-A+3G_{,r}\,(r_{\rm H})+\frac{2iqQ+6G(r_{\rm H})-4r_{{\rm H},v}}{r_{\rm H}}\right.$$
$$\left.-\frac{i\mu_0\lambda+\mu_0^2r_{\rm H}}{r_{\rm H}^2\left(\lambda^2+\mu_0^2r_{\rm H}^2\right)}[2G(r_{\rm H})-2r_{{\rm H},v}]\right\}\frac{\partial}{\partial r_*}R_2$$
$$=\left[\frac{A}{2\kappa}+2G(r_{\rm H})\right]\,\frac{\partial^2}{\partial r_*^2}R_2+2\,\frac{\partial^2}{\partial r_*\,\partial v_*}R_2$$
$$+\left[-A+3G_{,r}\,(r_{\rm H})+\frac{2iqQ+2G(r_{\rm H})}{r_{\rm H}}\right]\,\frac{\partial}{\partial r_*}R_2=0. \tag{20}$$

where we have used relations $2G(r_{\rm H})=2r_{{\rm H},v}$ and $\frac{\partial}{\partial r_*}R_1=0$.

With the aid of the event horizon equation (16), namely, $2G(r_{\rm H})=2r_{{\rm H},v}$, we know that the coefficient A is an infinite limit of $0/0$ type. By use of the L' Hôspital rule, we get the following result

$$A=\lim_{r\to r_{\rm H}(v_0)}\frac{2(G-r_{{\rm H},v})}{r-r_{\rm H}}=2G_{,r}\,(r_{\rm H}). \tag{21}$$

Now let us select the adjustable parameter κ in Eqs. (19) and (20) such that

$$\frac{A}{2\kappa}+2G(r_{\rm H})=\frac{G_{,r}\,(r_{\rm H})}{\kappa}+2r_{{\rm H},v}\equiv 1, \tag{22}$$

which means the temperature of the horizon is

$$\kappa = \frac{G,_r(r_{\rm H})}{1 - 2G(r_{\rm H})} = \frac{G, r(r_{\rm H})}{1 - 2r_{{\rm H},v}}. \tag{23}$$

Such a parameter adjustment can make Eqs. (19) and (20) reduce to

$$\frac{\partial^2}{\partial r_*^2} R_1 + 2\frac{\partial^2}{\partial r_* \partial v_*} R_1 = 0, \tag{24}$$

and

$$\begin{aligned} &\frac{\partial^2}{\partial r_*^2} R_2 + 2\frac{\partial^2}{\partial r_* \partial v_*} R_2 + \left[G,_r(r_{\rm H}) + \frac{2iqQ + 2G(r_{\rm H})}{r_{\rm H}}\right]\frac{\partial}{\partial r_*} R_2 \\ &= \frac{\partial^2}{\partial r_*^2} R_2 + 2\frac{\partial^2}{\partial r_* \partial v_*} R_2 + 2\,(C + i\omega_0)\,\frac{\partial}{\partial r*} R_2 = 0. \end{aligned} \tag{25}$$

where ω_0, C will be regarded as finite real constants,

$$\omega_0 = \frac{qQ}{r_{\rm H}}, \qquad 2C = G_{,r}(r_{\rm H}) + \frac{2r_{{\rm H},v}}{r_{\rm H}}.$$

Equations (24) and (25) are standard wave equations near the horizon.

5. THERMAL RADIATION SPECTRUM

Combining Eq. (24) with $\frac{\partial}{\partial r_*} R_1 = 0$, we know that R_1 is a constant near the horizon. The solution $R_1 = R_{10} e^{-i\omega v_*}$ means that Hawking radiation does not exist for R_1.

Now separating variables to Eq. (25) as

$$R_2 = R_2(r_*) e^{-i\omega v_*}$$

and substituting this into Eq. (25), one gets

$$R_2'' = 2[i(\omega - \omega_0) - C] R_2', \tag{26}$$

The solution is

$$R_2 = R_{21} e^{2[i(\omega-\omega_0)-C]r_*} + R_{20}. \tag{27}$$

The ingoing wave and the outgoing wave to Eq. (25) are

$$\begin{aligned} R_2^{\rm in} &= e^{-i\omega v_*}, \\ R_2^{\rm out} &= e^{-i\omega v_*} e^{2[i(\omega-\omega_0)-C]r_*}, \quad (r > r_{\rm H}). \end{aligned} \tag{28}$$

Near the event horizon, we have

$$r_* \sim \frac{1}{2\kappa} \ln(r - r_{\rm H}).$$

Clearly, the outgoing wave $R_2^{\rm out}(r > r_{\rm H})$ is not analytic at the event horizon $r = r_{\rm H}$, but can be analytically extended from the outside of the hole into the inside of the hole through the lower complex r plane

$$(r - r_{\rm H}) \to (r_{\rm H} - r)\, e^{-i\pi}$$

to

$$\tilde{R}_2^{\rm out} = e^{-i\omega v_*} e^{2[i(\omega-\omega_0)-C]r_*} e^{i\pi C/\kappa} e^{\pi(\omega-\omega_0)/\kappa}, \quad (r < r_{\rm H}). \tag{29}$$

So the relative scattering probability of the outgoing wave at the horizon is easily obtained

$$\left|\frac{R_2^{\rm out}}{\tilde{R}_2^{\rm out}}\right|^2 = e^{-2\pi(\omega-\omega_0)/\kappa}. \tag{30}$$

According to the method suggested by Damour and Ruffini (1976) and developed by Sannan (1988), the thermal radiation Fermionic spectrum of Dirac particles from the event horizon of the hole is given by

$$\langle \mathcal{N}_\omega \rangle = \frac{1}{e^{(\omega-\omega_0)/T_{\rm H}} + 1}, \tag{31}$$

with the Hawking temperature being

$$T_{\rm H} = \frac{\kappa}{2\pi},$$

whose obvious expression is

$$T_{\rm H} = \frac{1}{4\pi r_{\rm H}} \cdot \frac{M r_{\rm H} - Q^2 - \Lambda r_{\rm H}^4/3}{M r_{\rm H} - Q^2/2 - \Lambda r_{\rm H}^4/6}. \tag{32}$$

It follows that the temperature depends on the time, because it is determined by the surface gravity κ, a function of v. The temperature is consistent with that derived from the investigation of the thermal radition of Klein–Gordon particles (Li, 1998; Li *et al.*, 1999; Li and Zhao, 1998).

6. CONCLUSIONS

Equations (16) and (23) give the location and the temperature of event horizon, which depend on the advanced time v. They are just the same as that obtained in the discussion on thermal radition of Klein–Gordon particles in the same space–time. Equation (31) shows the thermal radiation spectrum of Dirac particles in a charged Vaidya black hole with a cosmological constant Λ.

In conclusion, we have studied the Hawking radiation of Dirac particles in a black hole whose mass and electric charge change with time. Our results are consistent with others. In this paper, we have dealt with the asymptotic behavior of the separated Dirac equation near the event horizon—not only its first order form but also its second order form. We find that the limiting form of its first order form puts very strong restrictions on the Hawking radiation, that is, not all components of Dirac spinors but P_2, Q_1 display the property of thermal radiation. The asymmetry of Hawking radiation with respect to the four-component Dirac spinors probably originate from the asymmetry of space–times in the advanced Eddington–Finkelstein coordinate. This point has not been revealed previously.

In addition, our analysis demonstrates that except the Coulomb energy ω_0, there was no new quantum effect in a Vaidya-type space–time as declared by Li (Li, 1998; Li *et al.*, 1999; Li and Zhao, 1998). This conclusion holds true in any spherically symmetric black hole whether it is static or nonstatic.

ACKNOWLEDGMENT

S. Q. Wu is indebted to Dr. Jeff at Motomola Company for his long-term help. This work was supported in part by the NSFC and Hubei-NSF in China.

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